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Preface

Biostatistics and statistics departments are struggling with how much probability and measure theory to include in their curricula. The traditional statistics department model of a full year of probability using the texts of Billingsley or Chung, for example, is losing favor as students are in a rush to get through a graduate program and begin their careers. Some biostatistics departments have gone to the extreme of eliminating graduate-level probability altogether. Consequently, their students are left with a background that does not prepare them to make rigorous arguments. We wrote this book as a compromise between the two unpalatable extremes: overloading statistics students with extensive and mathematically challenging measure theory versus leaving them unprepared to prove their results. Rather than offering a comprehensive treatment of all of probability and measure theory, replete with proofs, we present the essential probability results that are used repeatedly in statistics applications. We also selectively present proofs that make repeated use of mathematical techniques that we continue to use in our statistical careers when rigor is needed. As biostatisticians, we have encountered numerous applications requiring careful use of probability. We share these in this book, whose emphasis is on being able to understand rigorously the meaning and application of probability results. While traditional graduate probability books are sometimes written for mathematics students with no knowledge of elementary statistics, our book is written with statisticians in mind. For example, we motivate characteristic functions by first discussing harmonic regression and its usefulness in understanding circadian rhythm of biological phenomena. Another example of our use of statistical applications to help understand and motivate probability is the study of permutation tests. Permutation tests provide fertile ground for understanding conditional distributions and asymptotic arguments. For example, it is both challenging and instructive to try to understand precisely what people mean when they assert the asymptotic equivalence of permutation and t-tests. In summary, we believe that this book is ideal for teaching students essential probability theory to make rigorous probability arguments.

The book is organized as follows. The first chapter is intended as a broad introduction to why more rigor is needed to take that next step to graduate-level probability. Chapter 1 makes use of intriguing paradoxes to motivate the need for rigor. The chapter assumes knowledge of basic probability and statistics. We return to some of these paradoxes later in the book. Chapter 2, on countability, contains essential background material with which some readers may already be familiar. Depending on the background of students, instructors may want to begin with this chapter or have students review it on their own and refer to it when needed. The remaining chapters are in the order that we believe is most logical. Chapters 3 and 4 contain the backbone of probability: sigma-fields, probability measures, and random variables and vectors. Chapter 5 introduces and contrasts Lebesgue-Stieltjes integration with the more familiar Riemann-Stieltjes integration, while Chapter 6 covers different modes of convergence. Chapters 7 and 8 concern laws of large numbers and central limit theorems. Chapter 9 contains additional results on convergence in distribution, including the delta method, while Chapter 10 covers the extremely important topic of conditional probability, expectation, and distribution. Chapter 11 contains many interesting applications from our actual experience. Other applications are interspersed throughout the book, but those in Chapter 11 are more detailed. Many examples in Chapter 11 rely on material covered in Chapters 9-10, and would therefore be difficult to present much earlier. The book concludes with two appendices. Appendix A is a brief review of prerequisite material, while Appendix B contains useful probability distributions and their properties. Each chapter contains a chapter review of key results, and exercises are intended to constantly reinforce important concepts.

We would like to express our extreme gratitude to Robert Taylor (Clemson University), Jie Yang (University of Illinois at Chicago), Wlodek Byrc (University of Cincinnati), and Radu Herbei (Ohio State University) for reviewing the book. They gave us very helpful suggestions and additional material, and caught typos and other errors. The hardest part of the book for us was constructing exercises, and the reviewers provided additional problems and suggestions for those as well.

Index of Statistical Applications and Notable Examples

In this text, we illustrate the importance of probability theory to statistics. We have included a number of illustrative examples that use key results from probability to gain insight on the behavior of some commonly used statistical tests, as well as examples that consider implications for design of clinical trials. Here, we highlight our more statistical applications and a few other notable examples.

Statistical Applications

Other Notable Examples